Pension income indexation: a mean-variance approach

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Abstract

We propose a theoretical framework to study pension income indexation from the point of view of the pensioner. We model pensioner’s indexation measure choice in a mean-variance optimal portfolio framework. Similarly to the standard mean-variance portfolio choice problem, the optimal choice of the indexation measure depends not only on the return but also the associated risk. We consider two alternative risky assets and a risk free asset given by the pensioner specific inflation rate. We then use the theoretical model to calculate the optimal portfolio using historical data from Uruguay. We first approximate changes in the cost of living of pensioners by computing household specific inflation rates. We then compute the optimal portfolio in terms of the Average National Earnings Index (ANEI) and the Consumer Price Index (CPI) for different generations of pensioners using historical data. Our results suggest that for most of the cohorts but particularly for those retiring in the 70s and 80s, the optimal strategy is to choose the CPI as the pension income indexation measure. Even for cohorts retiring after the 1989 constitutional reform that established the ANEI as the indexation measure, the CPI is the preferred indexation measure. We also present two alternative criteria to assess the convenience of each indexation measured and arrive to similar results.

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Keywords: cost of living, pension income, indexation, mean-variance portfolio choice

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1 Introduction

The choice of which indexation measure to use to uprate pension income is of particular importance in periods of high inflation. Several countries have automatic indexation either using the Consumer Price Index (CPI) or the Average Nominal Earnings Index (ANEI) or even a combination of both.

In this paper we propose a theoretical framework to study pension income indexation from the point of view of the pensioner. We model pensioner's indexation choice in a mean-variance optimal portfolio framework. Similarly to the standard mean-variance portfolio choice problem, the optimal choice of the indexation measure depends not only on the return but also the associated risk. We consider two alternative risky assets and a risk free asset given by the pensioner specific inflation rate.

We then take the model to the data. According to article 67 of the Constitution and Law No. 17,687, pension income in Uruguay is adjusted by the previous year increase in the official ANEI or Indice Medio de Salarios Nominales. We first approximate changes in the cost of living of pensioners by computing household specific inflation rates. We then compute the optimal portfolio in terms of ANEI and CPI for different generations of pensioners using historical data.

The rest of the paper is organized as follows. The literature is summarized in Section 2. Section 3 present the mean-variance portfolio choice model and the model is taken to the data in Section 4. Finally, Section 5 concludes.

2 Literature Review

A strand of literature in the UK has focused on the inflation experience of different household types: Crawford (1994), Crawford and Smith (2002), Leicester et al. (2008) and Levell and Oldfield (2011). Crawford (1994) estimates Tornqvist type price indexes for 74 commodities to study changes in the cost of living of different household types during the period 1978 to 1992. He finds small differences in the inflation rate experienced by the different types of households, in particular, richer households experienced higher inflation than poorer during this period due to the fall in relative prices of necessities and the corresponding increase of luxuries.

Crawford and Smith (2002) study the inflation experience of different types of households during the period 1976 to 2000. Using data from the UK Family Expenditure Survey (FES) and computing household specific inflation rates, they find that the distribution of inflation varies substantially over time. They do
not find a particular pattern in the dispersion of inflation across households over time but suggest that household inflation is more dispersed in periods of high inflation. They then study how representative is the average rate of inflation finding that, between 1976 and 2000, only 35 percent of the households experience inflation within 1 percentage point of the average. They also study the inflation experience of different types of households finding that, on average, inflation is higher for high income households, non-pensioners, mortgagors, employed, single adults and younger. Finally, they show the importance of allowing differential effects of inflation in studying inequality over time.

More recently, Leicester et al. (2008) study the inflation experience of older households and Levell and Oldfield (2011) the inflation experience of low-income households in the UK. Of particular interest for this study is the analysis of Leicester et al. (2008). They use data from the UK FES to compute household specific inflation between 1977 and 2008 and find that, albeit substantial differences in given years, during the whole period there is no difference in the average inflation rate of pensioners (5.8 percent) and non-pensioners (5.9 percent). They then study how inflation varies within pensioners and find that those aged 75 or above suffered more from the rapid increase in fuel and food prices than younger pensioners.

3 Model: mean-variance portfolio choice

Assume a worker retires at time \( t = 0 \) and start receiving her pension income that is adjusted on a yearly basis. The aim of adjusting pension income is to maintain pensioners’ cost of living. If the pensioner has the option, which measure would she choose? A pensioner maximizing her intertemporal utility of consumption will choose the indexation measure that allows her to smooth consumption over time. Assuming the pensioner has no assets to finance her consumption, pension income - her only source of income - has to be adjusted by a pension specific cost of living index in order to allow her to maintain consumption over time. Let’s call this pension specific cost of living index \( p^* \).

Let\( r_p^r = r_p^{p^*} + c_p^p \) be the rate of return of \( p \) at time \( t \) and \( r_s^r = r_s^{p^*} + c_s^p \) the rate of return of \( s \). We can express these rate of returns with respect to the return of the risk free asset given by the pension specific
cost of living index, \( r_t^\nu \). Thus, we define the excess returns for \( p \) and \( s \) respectively as:

\[
\tilde{r}_t^p = r_t^p - r_t^\nu = \epsilon_t^p
\]

\[
\tilde{r}_t^s = r_t^s - r_t^\nu = \epsilon_t^s
\]

Assume that:

\[
\left( \begin{array}{c} \epsilon_t^p \\ \epsilon_t^s \end{array} \right) \sim \left[ \left( \begin{array}{c} \mu_t^p \\ \mu_t^s \end{array} \right), \left( \begin{array}{cc} \sigma_p^2 & Cov(\epsilon_t^p, \epsilon_t^s) \\ Cov(\epsilon_t^p, \epsilon_t^s) & \sigma_s^2 \end{array} \right) \right]
\]

Thus, we assume that whilst the expected mean excess return changes overtime the variance and co-

variance are fixed. This implies that the excess return distribution shapes are assumed to remain the same
during retirement but translates with the mean.

We can think of the pensioner selection problem as a mean-variance portfolio selection problem in which
the pensioner receives a lump sum at time 0 and then chooses the proportion of income to allocate to each
asset. We then assume that the pensioner chooses the proportion of income allocated to each indexation
measure at the time of retirement and that proportion is kept constant until she dies at time \( T \). Let \( \gamma \) be the
coefficient of relative risk aversion and \( \beta = \frac{1}{1+\delta} \) the subjective discount factor, then, the pensioner chooses
\( w_p \) by solving the following optimization problem:

\[
\text{Max}_{w_p} \quad V = E_0 \left[ \sum_{t=1}^{T} \beta^t \epsilon_t^p w_p + (1 - w_p)\epsilon_t^s \right] - \frac{\gamma}{2} \left[ \sum_{t=1}^{T} \beta^t \text{Var}[\epsilon_t^p w_p + (1 - w_p)\epsilon_t^s] \right]
\]

\[
\text{st} \quad 0 \leq w_p \leq 1
\]

Given the assumptions on the distribution of returns, the mean and variance can be expressed as follows:

\[
E_0 \left[ \sum_{t=1}^{T} \beta^t \epsilon_t^p w_p + (1 - w_p)\epsilon_t^s \right] = \sum_{t=1}^{T} \beta^t [\mu_t^p w_p + (1 - w_p)\mu_t^s]
\]

\[
\text{Var}[\epsilon_t^p w_p + (1 - w_p)\epsilon_t^s] = \sum_{t=1}^{T} \beta^t [\sigma_t^2 w_p + (1 - w_p)\sigma_t^2]
\]
\[
\sum_{t=1}^{T} \beta^t \text{Var}[\epsilon_t^p w_p + (1 - w_p) \epsilon_t^s]
= \sum_{t=1}^{T} \beta^t [\sigma_p^2 w_p^2 + (1 - w_p)^2 \sigma_s^2 + 2 w_p (1 - w_p) \text{Cov}(\epsilon_t^p, \epsilon_t^s)]
\]

Plugging in these expressions, the pensioner problem can thus be expressed as:

\[
\begin{align*}
\text{Max}_{0 \leq w_p \leq 1} \quad V &= \sum_{t=1}^{T} \beta^t [\mu_t^p w_p + (1 - w_p) \mu_t^s] \\
&\quad - \frac{\gamma}{2} \left[ \sum_{t=1}^{T} \beta^t [\sigma_p^2 w_p^2 + (1 - w_p)^2 \sigma_s^2 + 2 w_p (1 - w_p) \text{Cov}(\epsilon_t^p, \epsilon_t^s)] \right]
\end{align*}
\]

The first order condition with respect to \( w_p \) is:

\[
\frac{dV}{dw_p} = \sum_{t=1}^{T} \beta^t [\mu_t^p - \mu_t^s] \\
- \gamma w_p \left[ \sum_{t=1}^{T} \beta^t [\sigma_p^2 + \sigma_s^2 - 2 \text{Cov}(\epsilon_t^p, \epsilon_t^s)] \right] \\
+ \gamma \left[ \sum_{t=1}^{T} \beta^t [\sigma_s^2 - \text{Cov}(\epsilon_t^p, \epsilon_t^s)] \right] = 0
\]

Define \( \mu_i = \sum_{t=1}^{T} \beta^t \mu_t^i \) for \( i = p, s \). Then,

\[
w_p^* = \frac{\tilde{\mu}_p - \tilde{\mu}_s + \gamma \sum_{t=1}^{T} \beta^t [\sigma_p^2 + \sigma_s^2 - 2 \text{Cov}(\epsilon_t^p, \epsilon_t^s)]}{\gamma \sum_{t=1}^{T} \beta^t [\sigma_p^2 + \sigma_s^2 - 2 \text{Cov}(\epsilon_t^p, \epsilon_t^s)]} \quad (3.3)
\]

Equation 3.3 gives the expression for the optimal proportion of income allocated to the CPI indexation measure and thus \( (1 - w_p^*) \) represents the optimal proportion of income allocated to the ANEI indexation measure.

We use the theoretical model to calculate the optimal portfolio using historical data from Uruguay in the following sections.

4 The case of Uruguay

Pension income in Uruguay is indexed to the ANEI. Indeed, according to article 67 of the Constitution and Law No. 17,687, pension income in Uruguay is adjusted by the previous year increase in the official average nominal earnings index (ANEI) or Indice Medio de Salarios Nominales (IMSN). In this section
we assess, based on the theoretical model, the convenience, from the point of view of the pensioners, of uprating pension income using the ANEI or the CPI. We then assume that the pensioners have access to two alternative assets and have to decide the proportion of income allocated to each of them.

4.1 Preliminaries: household inflation

Note that as the excess returns are expressed with respect to the cost of living of pensioners we first have to compute a pensioner specific inflation index \( p^* \). Define inflation rate for household \( i \) at time \( t \) as:

\[
p^*_t = \sum_{j=1}^{J} s^j_i \pi^j_t
\]

(4.1)

Where: \( s^j_i = \frac{p^j_i q^j_i}{p^j_i q^j_i} \), is the share of expenditure of household \( i \) in good \( j \) and \( \pi^j_t \) is the year-on-year inflation rate of good \( j \) at time \( t \). Inflation rates for the different goods are only available at the national level from the National Statistics Office (INE) and thus variation in the inflation experienced by different households is due to differences in the expenditure shares. To compute the household specific inflation rate we consider 80 sections of the CPI \( (j=1,\ldots,80) \) and compute \( s^j_i \) for each household in the sample using the 2005/06 Encuesta de Gastos e Ingresos de los Hogares. Household inflation depends then, on the CPI section inflation rate and on the basket of goods consumed.

Table 1 shows the consumption basket, as a proportion of total expenditure, for workers and retirees. Similar to previous studies, we find that pensioners spend a smaller proportion of their budget in work related goods, such as clothing and footwear, transport and eating out. We also find, as expected, that pensioners spend less in education than workers. On the other hand, the share of total expenditure that pensioners spend in healthcare almost double that of workers (6.4% vs 12.2%). Finally, pensioners also spend a larger proportion of their budget in housing.

These differences in the consumption bundle translate into differences in the inflation rate experienced by the different types of households. Figure 1 shows the inflation rate of workers and retirees and also the CPI based inflation rate calculated by the Office for National Statistics. First, our inflation measures follow quite closely the officially calculated inflation rate. The small differences are due to the fact that the official calculation make use of additional data sources besides the budget survey we use. A second feature that arises from the graph is the fact that there are not substantial differences in the inflation experienced by
Table 1: Consumption basket (in % of total expenditure)

<table>
<thead>
<tr>
<th></th>
<th>Worker</th>
<th>Retiree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and non-alcoholic drinks</td>
<td>23.9</td>
<td>23.4</td>
</tr>
<tr>
<td>Alcoholic drinks and tobacco</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>4.9</td>
<td>2.8</td>
</tr>
<tr>
<td>Housing</td>
<td>30.4</td>
<td>35.4</td>
</tr>
<tr>
<td>Furniture and household appliances</td>
<td>4.3</td>
<td>4.0</td>
</tr>
<tr>
<td>Healthcare</td>
<td>6.4</td>
<td>12.2</td>
</tr>
<tr>
<td>Transport</td>
<td>9.0</td>
<td>5.7</td>
</tr>
<tr>
<td>Communications</td>
<td>4.1</td>
<td>4.2</td>
</tr>
<tr>
<td>Leisure</td>
<td>5.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Education</td>
<td>1.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Restaurant and hotels</td>
<td>3.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Other goods and services</td>
<td>4.8</td>
<td>4.6</td>
</tr>
</tbody>
</table>

workers and retirees in the period under analysis.

Figure 1: Average household inflation vs CPI: 1998-2012

Figure 2 shows the difference in the inflation rate experienced by pensioners relative to the general population and the change in the ANEI. In general, pensioners experience slightly higher inflation rates in periods of economic distress as the one experienced after the Argentina devaluation of 2001-2002. With a drop of 9% in real GDP between 2001 and 2002, pensioners annual inflation rate reached almost 30% by the end of 2002. As it is also clear from the graph, using ANEI as the pension indexation measure does not insure pensioners consumption during recessions or periods of low economic growth. Indeed, pension income
- adjusted by the ANEI - purchasing power declined by about 28% during the 2001-2002 economic crisis.

Figure 2: Average household inflation difference and ANEI

![Graph showing average household inflation difference and ANEI](image)

In the next section we take the model to the data and show, using historical data series for Uruguay, pensioners optimal portfolio choices.

4.2 Cohorts

For the empirical analysis we work with cohort of retirees. We have data on the dynamics of the CPI, the ANEI and the pensioners specific inflation for the period 1968 to 2012. We then define a cohort based on the year of retirement. The length of retirement ($T$) is assumed to be 15 years$^1$. Then, someone retired in 1970, for instance, is assumed to live and earn pension income until 1985. Similarly, someone retired in 1980 is assumed to live and earn pension income until 1995. Given we have data until 2012, the last cohort we consider is the one retiring in 1997.

Figure 3 shows pension income dynamics using either the CPI or the ANEI as indexation measures for four cohorts: 1970, 1980, 1990 and 1997. Based just on pension income obtained in the last year of retirement it is clear that the year of retirement is of major importance to choose which indexation measure makes the pensioner better off. The 1970 cohort is better off during the whole retirement period by having its pension income adjusted by the CPI. On the other hand, the 1997 cohort is better off at the beginning.

$^1$Similar results are obtained if we consider, for instance, $T=18$ shown in the Appendix. Note that life expectancy at 60, the normal retirement age, is about 16 years for men and 18 years for women.
of retirement with the ANEI, with the CPI after the 2002 crisis and then with the ANEI by the end of the retirement period. The theoretical model presented above is one way to summarize the trajectory of pension income dynamics in one measure. Alternative summary measures are presented as robustness checks in Section 5.1.

Figure 3: Pension income dynamics under alternative indexation measures (in logs)

In the following section we present the inputs for the computation of $w_p^*$. 

4.3 Return and risk

Having computed pensioners specific inflation ($p^*$), we can calculate the excess returns given in equations 3.1 and 3.2 that serve as inputs for the computation of the optimal proportion of income allocated to the CPI, $w_p^*$; given in equation 3.3.

Figure 4 shows the expected returns - $\mu_p^*$ y $\mu_s^*$ - calculated as the annual percentage change in the CPI and ANEI respectively. As expected, the general population CPI is closer to the pensioners inflation rate than the ANEI. Due to the re-installation of the collective wage bargaining in 2005 together with the steady economic growth after the 2001-2002 economic crisis, nominal wages grew faster than inflation. Focusing on the period from 2000 until the last year of the series, it is clear that, with the exception of the drop in real wages in 2002 and without considering income risk, the pensioners would be better off investing in the ANEI. We show in the next sections that once we consider the variance of returns and the pensioners level
of risk aversion this result can change.

Figure 4: Return: $\mu_p^t$ $\gamma$ $\mu_s^t$

Figure 5 shows the discounted accumulated expected returns for each cohort of retirees. For the cohorts retiring between 1969 and 1978 and those retiring between 1987 and 1994 - with the exception of 1990 -, $\tilde{\mu}_p > \tilde{\mu}_s$. On the other hand, for those retiring between 1979 and 1986, in 1990 and between 1995 and 1997, $\tilde{\mu}_p < \tilde{\mu}_s$. A second important feature of the graph is the fact that for both assets but in particular for the ANEI, the discounted accumulated return increases over time. Moreover, $\tilde{\mu}_s > 0$ only for those retiring in 1997, the last cohort considered in the empirical analysis.

Figure 5: Return: $\tilde{\mu}_p$ $\gamma$ $\tilde{\mu}_s$
Figure 6 shows the implied income risk, measured by the variance of returns, under the two indexation measures. The risk associated to the ANEI is on average, about 55 times that of the CPI and ranges from a minimum of 15 times for the 1986 cohort to 108 times for the 1972 cohort. This implies that investing in the ANEI involves substantially more risk than investing in the CPI. An important point to highlight is the fact that, together with the increase in the expected return over time, the risk associated with the ANEI declines over time.

Figure 6: Pension income risk: $\sigma_p^2$ vs $\sigma_s^2$

5 Optimal $w_p$

Having presented the inputs to compute the optimal proportion of income to allocate to the CPI, we show in Figure 7 the results for each of the cohorts retiring between 1968 and 1997. A value of 1 implies that all the income should be invested in the CPI whilst a value of 0 implies that all the income should be invested in the ANEI. The results are shown for coefficients of risk aversion ($\gamma$) equal to 0.9, 1.5 and 2.5. Note that given the much smaller risk associated with investing in the CPI, for a given level of return, $w_p^*$ increases with $\gamma$. This implies that if $w_p=1$ for $\gamma=0.9$, then it is also 1 for $\gamma=1.5$ and $\gamma=2.5$. The graph then shows in red and green the increase in $w_p^*$ due to an increase in the pensioner risk aversion. Take the case of the 1979 cohort. For a low level of risk aversion, the optimal strategy would be to invest all the income in the

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2These values were taken from Friedberg (2000) and particularly Chetty (2006) who estimates implied coefficients of relative risk aversion for older men aged 63 to 71.
ANEI. As risk aversion increases, $w_p^*$ also increases and thus $w_p^*$ is 0.4 for $\gamma=1.5$ and 0.6 for $\gamma=2.5$.

Our results suggest that for most of the cohorts but particularly for those retiring in the 70s and 80s, the optimal strategy is to choose the CPI as the pension income indexation measure. Even for cohorts retiring after the 1989 constitutional reform that established the ANEI as the indexation measure, the CPI is the preferred indexation measure.

Figure 7: Optimal $w_p^*$: sensitivity to risk aversion

5.1 Robustness check

In the previous sections we presented a framework to analyze the convenience of the CPI or the ANEI as pension income indexation measures. In this section we show that the results using the theoretical model presented in Section 3 are robust to alternative criteria. In particular, Figure 8 shows the log difference between pension income uprated by the ANEI and the CPI. Negative values implies that the CPI is preferred to the ANEI. Most of the cohorts in most of the years are better off if their pension income is adjusted by the CPI as opposed to the ANEI. Even for some of the cohorts that experience the years after the 2001 crisis in which the ANEI increased faster than the CPI are better off with the CPI. This is due to the negative effect that the 2001 crisis had on the real growth of wages and suggest that extreme events have profound effects in the convenience, from the point of view of the pensioner, of one or other pension income indexation measure.
A second criterion to assess the convenience of the CPI and ANEI is the Net Present Value (NPV). We assume a discount rate equal to 3% and show in Figure 9 the log difference of the NPV using the ANEI and the CPI. Again, negative values implies that the pensioner is better off with the CPI as opposed to the ANEI. With the exception of the 1979 cohort, cohorts between 1982 and 1986 and the 1990 cohort, the CPI is preferred to the ANEI.
6 Conclusion

We propose a theoretical framework to study pension income indexation from the point of view of the pensioner. We model pensioner’s indexation measure choice in a mean-variance optimal portfolio framework. Similarly to the standard mean-variance portfolio choice problem, the optimal choice of the indexation measure depends not only on the return but also the associated risk. We consider two alternative risky assets and a risk free asset given by the pensioner specific inflation rate.

We then use the theoretical model to calculate the optimal portfolio using historical data from Uruguay. According to article 67 of the Constitution and Law No. 17,687, pension income in Uruguay is adjusted by the previous year increase in the official ANEI or Indice Medio de Salarios Nominales. We first approximate changes in the cost of living of pensioners by computing household specific inflation rates. We then compute the optimal portfolio in terms of ANEI and CPI for different generations of pensioners using historical data. Our results suggest that for most of the cohorts but particularly for those retiring in the 70s and 80s, the optimal strategy is to choose the CPI as the pension income indexation measure. Even for cohorts retiring after the 1989 constitutional reform that established the ANEI as the indexation measure, the CPI is the preferred indexation measure. We also present two alternative criteria to assess the convenience of each indexation measured and arrive to similar results.
7 Appendix

Figure 10: Optimal $w_p^*$ with $T=18$: sensitivity to risk aversion

Figure 11: Optimal $w_p^*$ with $T=19$: sensitivity to risk aversion
Figure 12: Optimal $w_p^*$ with $T=20$: sensitivity to risk aversion